

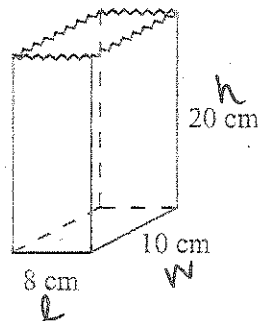
1.3B SURFACE AREA: Nets of Composite Objects

Surface Area of Cubes and Rectangular Solids

The surface area of a solid is the sum of the area of all its faces. Some examples are: the amount of material to build a house, or the amount of material to make a cardboard box, or to make a tin can.



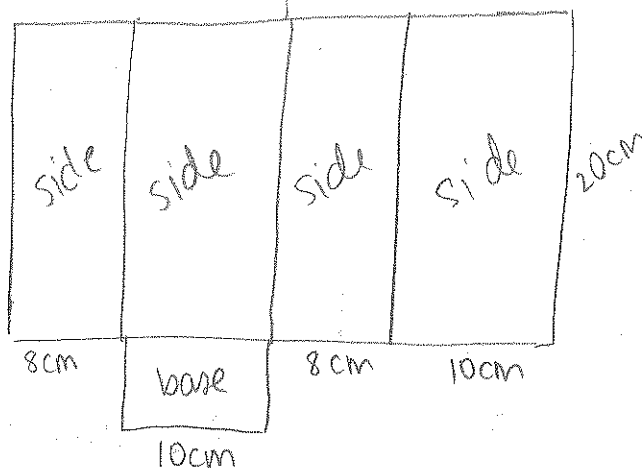
Consider a paper bag. How much paper is needed to make the bag?



► Solution: Draw a NET for the bag

A net is a 2-D figure of the component parts of the 3-D figure

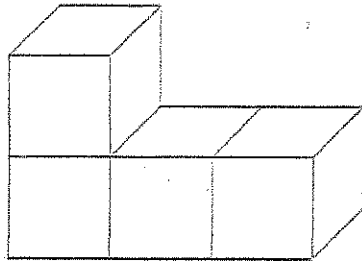
* there are 5 faces for a paper bag



$$\begin{aligned}
 SA &= 2lw + 2lh + 2wh \\
 &= 1 \cdot 8 \cdot 10 + 2 \cdot 8 \cdot 20 + 2 \cdot 10 \cdot 20 \\
 &= 80 + 320 + 400 \\
 &= \boxed{800 \text{ cm}^2}
 \end{aligned}$$

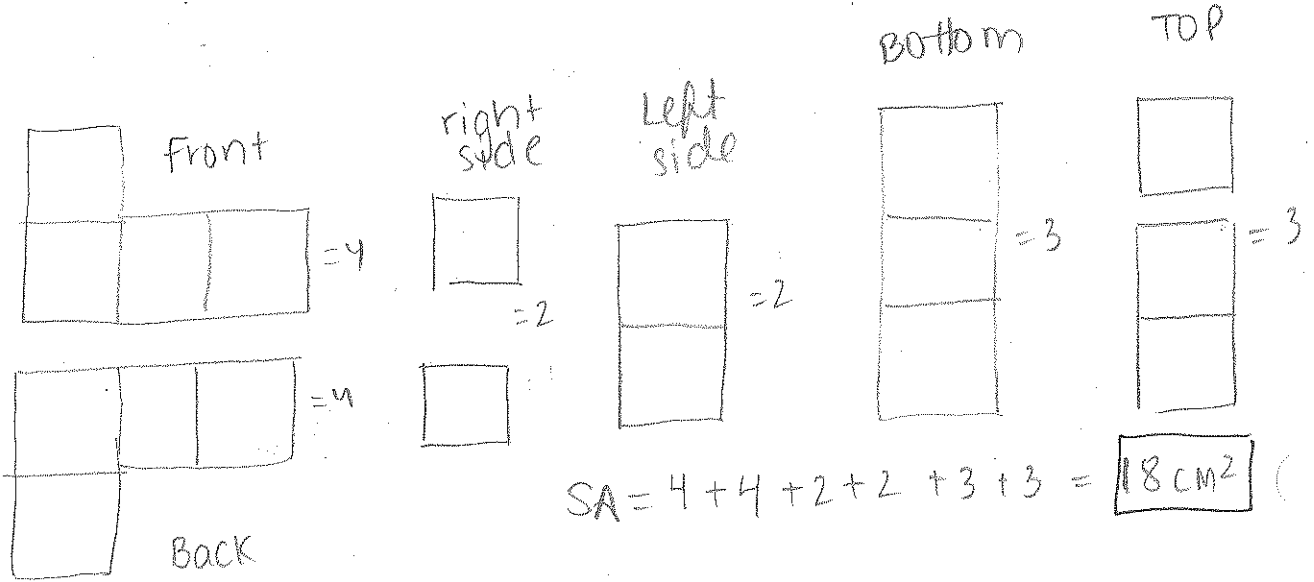


Determine the surface area of the composite of cubes.

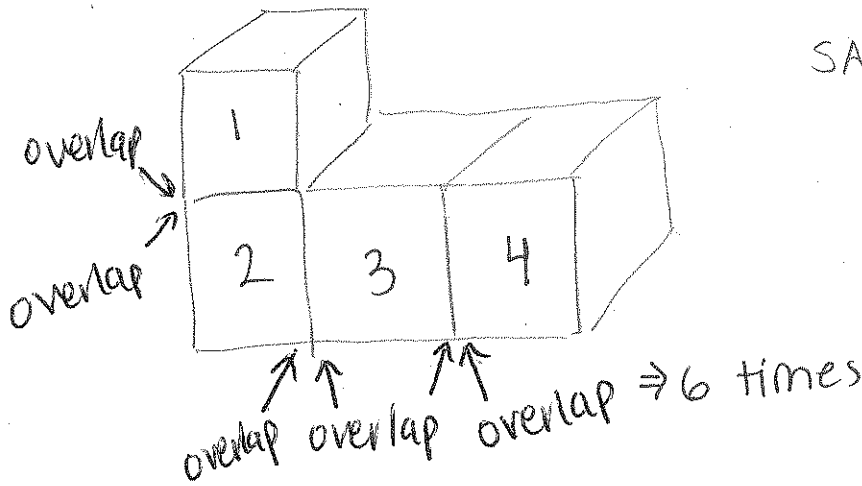


► Solution:

Method 1: Draw a net



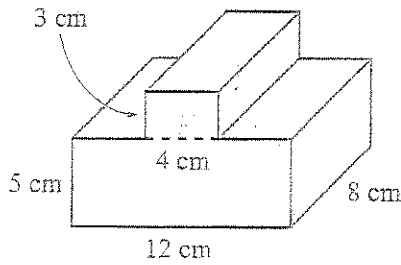
Method 2: consider a cube



$$\begin{aligned}
 \text{SA of 1 cube} &= 6a^2 \\
 &= 6 \cdot 1^2 \\
 &= 6 \cdot 1 = 6 \text{ cm}^2 \\
 &\quad \times 4 \text{ cubes} \\
 &= 24 \text{ cm}^2 \\
 &\quad - 6 \text{ cm}^2 \\
 \hline
 &= 18 \text{ cm}^2
 \end{aligned}$$

Example:

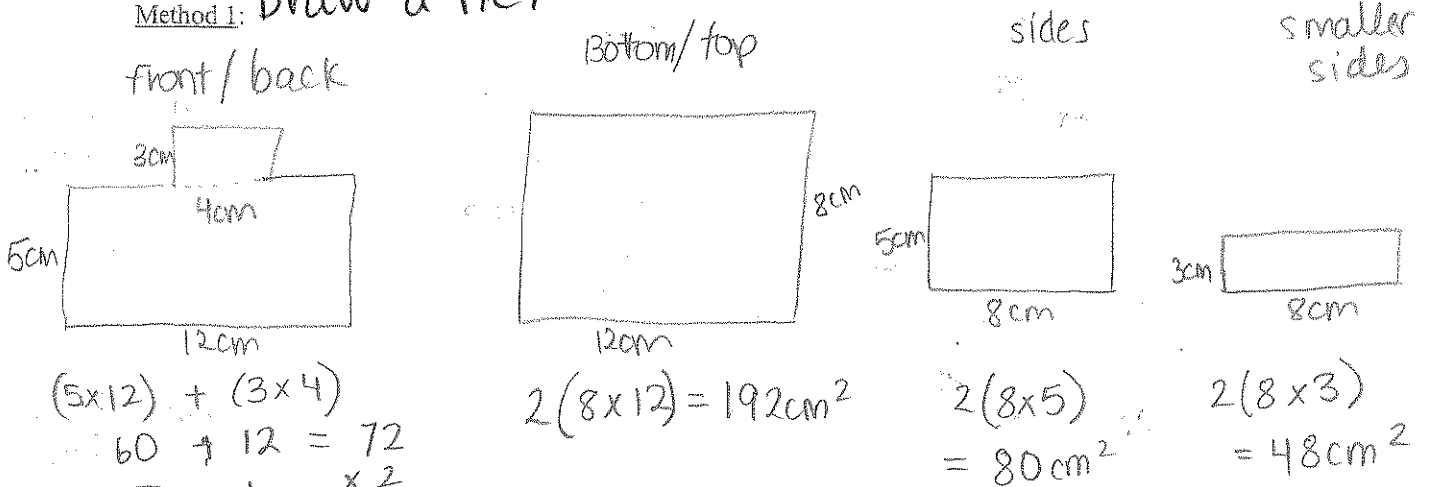
Determine the surface area of the composite figure.



$$144 + 192 + 80 + 48 = \boxed{464 \text{ cm}^2}$$

► Solution:

Method 1: Draw a net



$$(5 \times 12) + (3 \times 4) \\ 60 + 12 = 72 \\ \times 2 \\ \hline 144 \text{ cm}^2$$

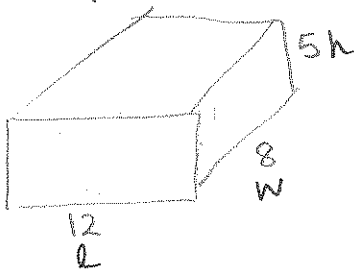
$$2(8 \times 12) = 192 \text{ cm}^2$$

$$2(8 \times 5) \\ = 80 \text{ cm}^2$$

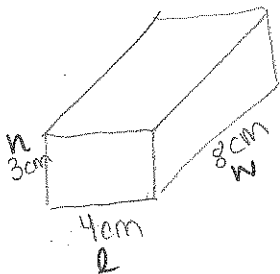
$$2(8 \times 3) \\ = 48 \text{ cm}^2$$

Method 2:

→ 2 rectangular solids minus OVERLAP



$$\begin{aligned} SA &= 2lw + 2lh + 2wh \\ &= 2 \cdot 12 \cdot 8 + 2 \cdot 12 \cdot 5 + 2 \cdot 8 \cdot 5 \\ &= 192 + 120 + 80 = 392 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} SA &= 2 \cdot 4 \cdot 8 + 2 \cdot 4 \cdot 3 + 2 \cdot 3 \cdot 8 \\ &= 64 + 24 + 48 = 136 \text{ cm}^2 \end{aligned}$$

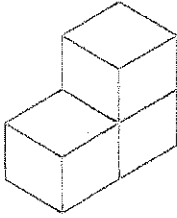
overlap: $2(8 \times 4) = 64 \text{ cm}^2$

$$\begin{aligned} &392 \\ &+ 136 \\ &- 64 \end{aligned}$$

$$\boxed{464 \text{ cm}^2}$$

1.) Determine the surface area of the composite of cubes. Each cube has sides of 1 unit.

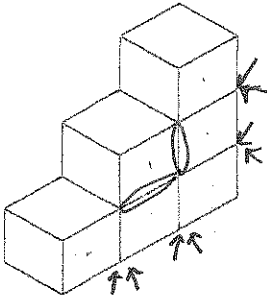
a)



$$\begin{aligned} \text{SA of 1 cube} &= 6(1^2) \\ &= 6 \\ &\times 3 \text{ cubes} \\ \hline 18 - 4 &= 14 \\ &\text{(overlap)} \end{aligned}$$

14 square units

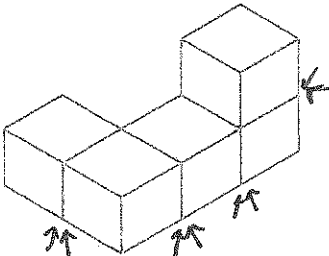
b)



$$\begin{aligned} \text{SA} &= 6(1^2) \\ &= 6 \\ &\times 6 \text{ cubes} \\ \hline 36 - 12 &= 24 \end{aligned}$$

24 square units

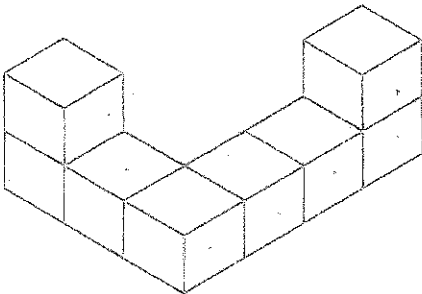
c)



$$\begin{aligned} \text{SA} &= 6 \\ &\times 5 \text{ cubes} \\ \hline 30 - 8 &= \end{aligned}$$

22 square units

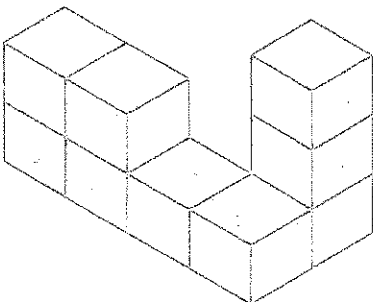
d)



$$\begin{aligned} \text{SA} &= 6 \\ &\times 8 \text{ cubes} \\ \hline 48 - 14 &= \end{aligned}$$

34 square units

e)



$$\begin{aligned} \text{SA} &= 6 \\ &\times 9 \text{ cubes} \\ \hline 54 - 18 &= \end{aligned}$$

36 square units

2.) Determine the surface area of the composite figure. (All measurements are in centimeters)

a)

$SA = 6a^2 \Rightarrow 6(6^2) = 6 \cdot 36 = 216$
 $SA = 2(3 \cdot 6 + 6 \cdot 6 + 6 \cdot 3)$
 $= 2(18 + 36 + 18)$
 $= 144$
 overlap: $2(6 \times 3) = 36$

216
 $+ 144$
 $- 36$

 324 cm^2

b)

$SA = 2(4 \cdot 8 + 4 \cdot 4 + 8 \cdot 4)$
 $= 2(32 + 16 + 32) = 160$
 $SA = 2(12 \cdot 8 + 12 \cdot 4 + 8 \cdot 4)$
 $= 2(96 + 48 + 32) = 352$
 overlap: $2(8 \times 4) = 64$

160
 $+ 352$
 $- 64$

 448 cm^2

c)

$SA = 6a^2 = 6(5^2) = 6 \cdot 25 = 150$
 $SA = 2(16 \cdot 10 + 10 \cdot 4 + 16 \cdot 4)$
 $= 2(160 + 40 + 64) = 528$
 overlap: $2(5 \times 5) = 50$

150
 $+ 528$
 $- 50$

 628 cm^2

d)

$SA = 2(10 \cdot 6 + 6 \cdot 6 + 10 \cdot 6)$
 $= 2(60 + 36 + 60) = 312$
 $= 312 - 2 \text{ hollow faces} + \text{hollow tube}$
 $= 312 - [2(4 \times 4)] + [4(4 \times 10)]$
 $= 312 - 32 + 160$
 $= 440$

440 cm^2

e)

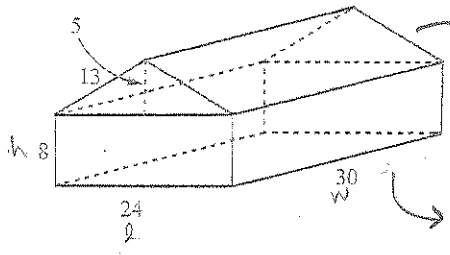
overlap: $= 2(1 \times 1) = 2$
 overlap: $= 2(2 \cdot 2) = 2 \cdot 4 = 8$
 $SA = 6a^2 = 6(1^2) = 6$
 $SA = 2(2 \cdot 2 + 2 \cdot 1 + 2 \cdot 1)$
 $= 2(4 + 2 + 2)$
 $= 2(8)$
 $= 16$
 $SA = 2(3 \cdot 1 + 3 \cdot 1 + 3 \cdot 3)$
 $= 2(3 + 3 + 9)$
 $= 2(15)$
 $= 30$

6
 $+ 16$
 $+ 30$
 $- 10$

 42 cm^2

- 3.) Find the surface area of the following figures. All measurements are in centimetres. All prisms are right prisms. Round answers to one decimal place.

a)



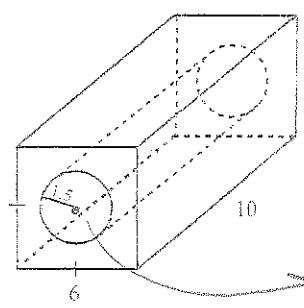
$$\begin{aligned}
 SA &= bh + 2ls + lb \\
 &= 24 \cdot 5 + 2 \cdot 30 \cdot 13 + 30 \cdot 24 \\
 &= 120 + 780 + 720 \\
 &= 1620
 \end{aligned}$$

$$\begin{array}{r}
 1620 \\
 + 2304 \\
 - 1440 \\
 \hline
 2484 \text{ cm}^2
 \end{array}$$

$$\begin{aligned}
 SA &= 2wh + 2lw + 2lh \\
 &= 2 \cdot 30 \cdot 8 + 2 \cdot 24 \cdot 30 + 2 \cdot 24 \cdot 8 \\
 &= 480 + 1440 + 384 \\
 &= 2304
 \end{aligned}$$

overlap: $2(24 \times 30)$
 $= 1440$

b)

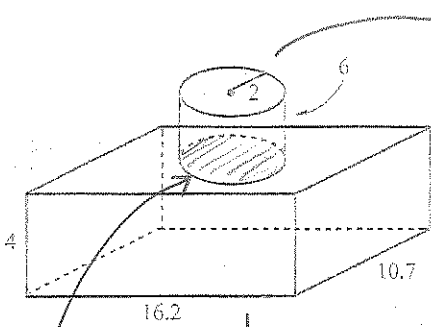


$$\begin{aligned}
 SA &= 2(lw + wh + lh) \\
 &= 2(10 \cdot 6 + 6 \cdot 6 + 10 \cdot 6) \\
 &= 2(60 + 36 + 60) \\
 &= 312
 \end{aligned}$$

$$\begin{array}{r}
 312 \\
 + 94.24 \\
 - 14.14 \\
 \hline
 392.1 \text{ cm}^2
 \end{array}$$

$$\begin{aligned}
 SA &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi(1.5^2) + 2\pi \cdot 1.5 \cdot 10 \\
 &= \cancel{14.14} + 94.24 \\
 &= 94.24
 \end{aligned}$$

c)



$$\begin{aligned}
 SA &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi 2^2 + 2\pi \cdot 2 \cdot 6 \\
 &= 2\pi 4 + 2\pi 12 \\
 &= 8\pi + 24\pi \\
 &= 32\pi \\
 &= 100.5309\dots
 \end{aligned}$$

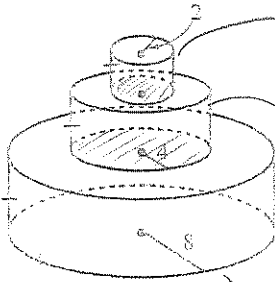
$$\begin{array}{r}
 100.5309\dots \\
 + 561.88 \\
 - 25.1327\dots \\
 \hline
 637.25 \text{ cm}^2
 \end{array}$$

overlap:

$$\begin{aligned}
 &= 2(\pi r^2) \\
 &= 2(\pi \cdot 2^2) \\
 &= 2(4\pi) \\
 &= 25.1327
 \end{aligned}$$

$$\begin{aligned}
 SA &= 2(4 \cdot 16.2 + 10.7 \cdot 16.2 + 4 \cdot 10.7) \\
 &= 2(64.8 + 173.34 + 42.8) \\
 &= 2(280.94) \\
 &= 561.88
 \end{aligned}$$

d)



$$\begin{aligned}
 SA &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi 2^2 + 2\pi \cdot 2 \cdot 4 \\
 &= 8\pi + 16\pi \\
 &= 25.1327 + 50.26548\dots \\
 &= 75.3982\dots
 \end{aligned}$$

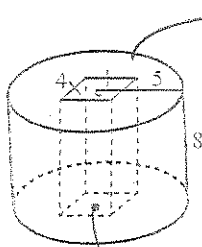
$$\begin{array}{r}
 75.3982\dots \\
 + 201.0619\dots \\
 + 603.185\dots \\
 - 125.6636\dots \\
 \hline
 754 \text{ cm}^2
 \end{array}$$

$$\begin{aligned}
 SA &= 2\pi 4^2 + 2\pi \cdot 4 \cdot 4 \\
 &= 32\pi + 32\pi \\
 &= 100.5309\dots + 100.5309\dots \\
 &= 201.0619\dots
 \end{aligned}$$

$$\begin{aligned}
 SA &= 2\pi 8^2 + 2\pi \cdot 8 \cdot 4 \\
 &= 128\pi + 64\pi \\
 &= 402.1238\dots + 201.0619\dots \\
 &= 603.185\dots
 \end{aligned}$$

$$\begin{array}{r}
 25.1327\dots \\
 + 100.5309\dots \\
 \hline
 125.6636\dots
 \end{array}$$

e)

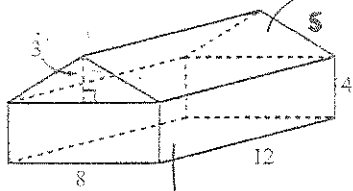


$$\begin{aligned}
 SA &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi 5^2 + 2\pi \cdot 5 \cdot 8 \\
 &= 50\pi + 80\pi \\
 &= 130\pi \\
 &= 408.407\dots
 \end{aligned}$$

$$\begin{array}{r}
 408.407\dots \\
 + 128 \\
 - 32 \\
 \hline
 504.4\text{ cm}^2
 \end{array}$$

$$\begin{aligned}
 SA &= 2(lw + lh + wh) \\
 &= 2(8 \cdot 4 + 8 \cdot 4 + 4 \cdot 4) \\
 &= 2(32 + 32 + 16) \\
 &= 64 + 64 + 32 \\
 &= 128
 \end{aligned}$$

f)



$$\begin{aligned}
 SA &= bh + 2ls + 2lw \\
 &= 8 \cdot 3 + 2 \cdot 12 \cdot 5 + 2 \cdot 12 \cdot 8 \\
 &= 24 + 120 + 96 \\
 &= 144
 \end{aligned}$$

$$\begin{array}{r}
 144 \\
 + 256 \\
 \hline
 400\text{ cm}^2
 \end{array}$$

use
pythagorean
theorem:



$$\begin{aligned}
 3^2 + 4^2 &= c^2 \\
 9 + 16 &= c^2 \\
 25 &= c^2 \\
 5 &= c
 \end{aligned}$$

$$\begin{aligned}
 SA &= 2lw + 2lh + 2hw \\
 &= 2 \cdot 12 \cdot 8 + 2 \cdot 12 \cdot 4 + 2 \cdot 4 \cdot 8 \\
 &= 96 + 96 + 64 \\
 &= 256
 \end{aligned}$$

- 4) Ann wants to wallpaper the walls of her bedroom. The rectangular room is 12 ft. by 15 ft. with height of the wall 8 ft. high. If a roll of wallpaper is 3 ft. wide and 50 ft. long, how many rolls of wallpaper are needed? (can't buy part of a roll, and ignore doors and windows)

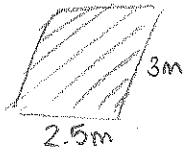


$$\begin{aligned} 4 \text{ walls only} &\Rightarrow SA = 2(8 \times 12) + 2(15 \cdot 8) \\ &= 192 + 240 \\ &= 432 \text{ ft.} \end{aligned}$$

$$\approx \boxed{3 \text{ rolls}}$$

$$1 \text{ roll} = 3 \times 50 = 150 \times 3 \text{ rolls} = 450 \text{ ft.}$$

- 5) Jessica wants to tile the kitchen floor with tiles that are 20 cm square. If the kitchen measures 2.5 m by 3 m, how many tiles are needed if 10% is added for waste?



$$2.5 \times 3 = 7.5 \text{ m}^2$$

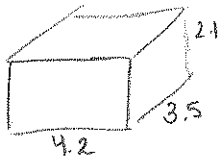
$$\begin{aligned} 20 \text{ cm} &\rightarrow 0.2 \text{ m} \rightarrow 0.2 \times 0.2 \\ &= 0.04 \text{ m}^2 \end{aligned}$$

$$7.5 \text{ m}^2 \div 0.04 \text{ m}^2 = 187.5 \text{ tiles} \times 1.1\% = 206.25$$

$$\begin{aligned} &\rightarrow 100\% \\ &+ 10\% \\ \hline &110\% \div 100\% = 1.1\% \end{aligned}$$

$$\approx \boxed{207 \text{ tiles}}$$

- 6) Andy has a contract to paint the walls and ceiling of 30 motel rooms each measuring 3.5 m by 4.2 m with height of 2.1 m. If a 4-litre pail of paint covers 40 m² and costs \$18.95, what is the cost of paint used?



$$\begin{aligned} SA &= 1(3.5 \cdot 4.2) + 2(4.2 \cdot 2.1) + 2(2.1 \cdot 3.5) \\ &= 14.7 + 17.64 + 14.7 \end{aligned}$$

$$= 47.04 \times 30 \text{ motel rooms} = \frac{1411.2 \text{ m}^2}{40 \text{ m}^2} = 35.28$$

$$\boxed{\$682.50}$$

$$\begin{aligned} &\times 36 \text{ pails} \\ &\times \$18.95 \\ \hline &\$682.50 \end{aligned}$$

- 7) What happens to surface area if you:

- a) Double each side of a cube?

x = edge of a cube

$$SA = 6x^2 \Rightarrow \text{doubled} \Rightarrow 2x \Rightarrow 6(2x)^2 = 6(4x^2) = 24x^2$$

becomes 4x larger

- b) Triple each side of a cube?

$$SA = 6x^2 \Rightarrow 6(3x)^2 = 6(9x^2) = 54x^2$$

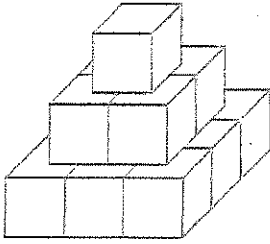
becomes 9x larger

- c) Leave one side the same length, double another side and triple the third side?

SA would double, then be \uparrow by a factor of $2lh + 8wh$

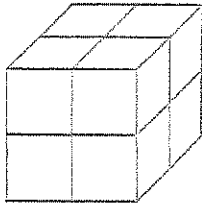
$3\frac{2}{3}$ times as large.

- 8) If you have 9^1 blocks and build a pyramid in the same pattern as the diagram shown, how many blocks are on the bottom layer?



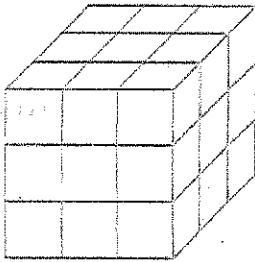
36

- 9) In the following diagram how many pieces have three sides showing?



8

- 10) In the following diagram how many pieces have:



a) Three sides showing?

8

b) Two sides showing?

12

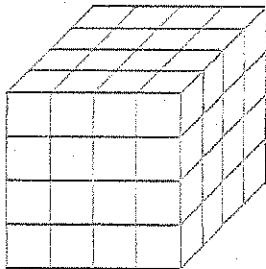
c) One side showing?

6

d) Zero sides showing?

1

- 11) In the following diagram how many pieces have:



a) Three sides showing?

8

b) Two sides showing?

24

c) One side showing?

16

d) Zero sides showing?

4