4.4 Fractional Exponents & Radicals

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RECALL: $a^m \cdot a^n = a^{m+n}$

To multiply:
↳ same base
↳ ADD exponents.

We can extend this law to powers with fractional exponents with numerator 1

POWERS With RATIONAL EXPONENTS with NUMERATOR 1

↳ n is a natural #
↳ x is a rational #

$$\frac{1}{x^n} = \sqrt[n]{x}$$

EX. #1

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5^{\frac{2}{2}} = 5^1 = \boxed{5} \checkmark$$

or...

$$\sqrt[2]{5^1} \cdot \sqrt[2]{5^1}$$

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = \boxed{5} \checkmark$$

Raising a # to the exponent $\frac{1}{2}$ is equivalent to taking the square root of the #

$$5^{\frac{1}{2}} = \sqrt[2]{5^1}$$

$$5^{\frac{1}{3}} = \sqrt[3]{5^1}$$

Ex. #2 Evaluate without using a calculator

(a.) $1000^{\frac{1}{3}}$

$= \sqrt[3]{1000^1}$

$= \sqrt[3]{10 \times 10 \times 10}$

$= \boxed{10}$

(b.) $0.25^{\frac{1}{2}}$

$= \sqrt[2]{0.25}$

$= \sqrt{0.5 \times 0.5} = \boxed{0.5}$

(c) $(-8)^{\frac{1}{3}}$

$= \sqrt[3]{-8^1}$

$= \sqrt[3]{-2 \times 2 \times 2}$

$= \boxed{-2}$

(d.) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$= \sqrt[4]{\left(\frac{16}{81}\right)^1} = \sqrt{\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}} = \boxed{\frac{2}{3}}$

Powers with Rational Exponents

↳ When m : n are natural #'s

↳ x is a rational #

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

and

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Ex. #3

$8^{\frac{2}{3}} = 8^{\frac{1}{3}\cdot 2}$

$= (8^{\frac{1}{3}})^{\cdot 2}$

$= (\sqrt[3]{8})^2$

$= (2)^2$

$= \boxed{4}$

$8^{\frac{2}{3}} = 8^{2\cdot\frac{1}{3}}$

$= (8^2)^{\frac{1}{3}}$

$= \sqrt[3]{(8^2)^1}$

$= \sqrt[3]{64}$

$= \boxed{4}$

Ex. #4

(a.) $0.01^{\frac{3}{2}}$

$= \sqrt{(0.01)^3}$

$= \sqrt{0.000001}$

$= \boxed{0.001}$

(b.) $(-27)^{\frac{4}{3}}$

$= \sqrt[3]{(-27)^4}$

$= \sqrt[3]{+531441}$

$= \boxed{81}$

(c.) $81^{\frac{3}{4}}$

$\sqrt[4]{81^3}$

$= \boxed{27}$

(d.) $0.75^{1.2}$

$= \boxed{0.7080\ldots}$

Exercises pg. 227 # 3-12, 15, 18