7.6 Properties of Systems of Linear Equations

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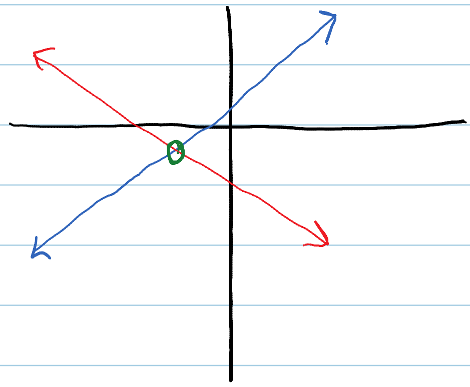
11:01 AM

All the linear systems you studied earlier have had exactly 1 solution.

↳ we graph a linear system to determine how many solutions it has.

Ex. #1

$$x + y = -3$$
$$-2x + y = 3$$

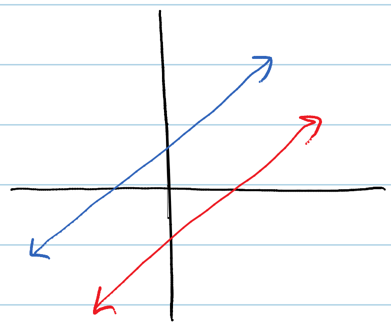


The graphs intersect @ (-2, -1) so there is only 1 solution

$x = -2$
$y = -1$

Ex. #2

$$-4x + 2y = 8$$
$$-2x + y = -2$$



The graphs don't intersect so there is NO solution because the slopes of the lines are equal; the lines are parallel

Ex. #3

$$2x + y = -1$$
$$4x + 2y = -2$$

Since the graphs overlap (coincide), every point on the lines are solutions → Infinite solutions

→ means unlimited or without bound

because the lines have equal slopes and the same y-intercept, they are coincident lines

Ex. #4 Determine the # of solutions of each linear system.

(a) $x + y = 3 \longrightarrow$ re-arrange it $y = -x + 3$ slope $= -1$, y-intercept $= 3$

$2x - y = -2 \longrightarrow y = -2x + 2$ slope $= -2$, y-intercept $= 2$

Slopes are different ∴ lines intersect @ exactly 1 point ∴ only 1 solution!

(b) $4x + 6y = -10 \longrightarrow$

$$\frac{\cancel{6}y}{\cancel{6}} = \frac{-4x}{6} - \frac{10}{6}$$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

$-2x - 3y = 5$

$$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{2x+5}{-3}$$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

The slope-intercept forms of both equations are the same, so the lines are coincident and the linear system has infinite solutions

(c) $2x - 4y = -1 \longrightarrow$

$$\frac{-\cancel{4}y}{-\cancel{4}} = \frac{-2x - 1}{-4}$$

$$y = \frac{1}{2}x + \frac{1}{4}$$

$3x - 6y = 2$

$$\frac{-\cancel{6}y}{-\cancel{6}} = \frac{-3x + 2}{-6}$$

$$y = \frac{1}{2}x - \frac{1}{3}$$

Slopes are equal! y-intercepts are different ∴ lines are parallel and the linear system has no solution

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