

PEARSON

Foundations and Pre-calculus Mathematics 10

Garry Davis
David Ellis
Jack Hope
Delcy Rolheiser
David Sufrin
Chris Van Bergeyk
David Van Bergeyk
David Zimmer

PEARSON

The Pearson logo consists of the word "PEARSON" in a blue, sans-serif font. Below the text is a yellow swoosh that starts under the 'P', goes under the 'A', and ends under the 'N'.

Publisher

Mike Czukar

Research and Communications Manager

Barbara Vogt

Design and Art Direction

Zena Denchik

Composition

Lapiz Digital Services, India

Photo Research

Carrie Gleason

Cover Image

Chad Baker/The Image Bank/GettyImages

Copyright © 2010 Pearson Canada Inc.

All rights reserved. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission, write to the Permissions Department.

ISBN-13 978-0-321-62684-4

ISBN-10 0-321-62684-2

Printed and bound in the United States

1 2 3 4 5 —CC— 14 13 12 11 10

Publishing Team

Enid Haley

Lesley Haynes

Claire Burnett

Stephanie Boragina

Ioana Gagea

Alison Rieger

Lynne Gulliver

Ellen Davidson

David Liu

May Look

Alison Dale

Cheri Westra

Carolyn Sebestyen

Jane Schell

Karen Alley

Judy Wilson

The information and activities presented in this book have been carefully edited and reviewed. However, the publisher shall not be liable for any damages resulting, in whole or in part, from the reader's use of this material.

Brand names that appear in photographs of products in this textbook are intended to provide students with a sense of the real-world applications of mathematics and are in no way intended to endorse specific products.

Advisors and Reviewers

Pearson Canada thanks its advisors and reviewers who helped shape the vision for *Foundations and Pre-calculus Mathematics 10* through discussions and reviews of prototype materials and manuscript.

Advisors

Bob Berglind

Calgary Board of Education
Calgary, AB

Marc Garneau

Surrey School District No. 36
Surrey, BC

Susan Ludwig

Edmonton Catholic Separate School District No. 7
Edmonton, AB

Christine Ottawa

Mathematics Consultant
Winnipeg, MB

Robert Sidley

Burnaby School District No. 41
Burnaby, BC

Francophone Advisors

Nathalie Houle

Winnipeg School Division
Winnipeg, MB

Danielle Lamoureux

Conseil scolaire centre-nord no. 2
Edmonton, AB

Northern Advisor

Peter Maguire

Yellowknife Education District No. 1
Yellowknife, NT

FNMI Advisors

Edward Doolittle

First Nations University of Canada
Regina, SK

Shirley A. Mykituk

Edmonton Catholic Separate School District No. 7
Edmonton, AB

Reviewers

Joe Beaudin

Calgary Board of Education
Calgary, AB

Rhonda German Hawthorne

St. James-Assiniboia School Division
Winnipeg, MB

Andrew Hirst

River East Transcona School Division
Winnipeg, MB

Terry Kaminski

Northern Lights School Division No. 69
Bonnyville, AB

Ryan Mann

Parkland School Division
Stony Plain, AB

Chad Raible

Maple Ridge-Pitt Meadows School District No. 42
Maple Ridge, BC

Ralph R. Schmidt

Elk Island Public Schools
Sherwood Park, AB

Contents

1 Measurement

1.1 Imperial Measures of Length	4
1.2 Math Lab: Measuring Length and Distance	13
1.3 Relating SI and Imperial Units	16
Checkpoint 1	24
1.4 Surface Areas of Right Pyramids and Right Cones	26
1.5 Volumes of Right Pyramids and Right Cones	36
1.6 Surface Area and Volume of a Sphere	45
Checkpoint 2	53
1.7 Solving Problems Involving Objects	55
Study Guide and Review	62
Practice Test	67

2 Trigonometry

2.1 The Tangent Ratio	70
2.2 Using the Tangent Ratio to Calculate Lengths	78
2.3 Math Lab: Measuring an Inaccessible Height	84
Checkpoint 1	87
2.4 The Sine and Cosine Ratios	89
2.5 Using the Sine and Cosine Ratios to Calculate Lengths	97
Checkpoint 2	103
2.6 Applying the Trigonometric Ratios	105
2.7 Solving Problems Involving More than One Right Triangle	113
Study Guide and Review	122
Practice Test	127
Project: Measurement <i>Ramp It Up!</i>	128
Cumulative Review: Chapters 1 and 2	130

3 Factors and Products

3.1 Factors and Multiples of Whole Numbers	134
3.2 Perfect Squares, Perfect Cubes, and Their Roots	142
Checkpoint 1	148
3.3 Common Factors of a Polynomial	150
3.4 Math Lab: Modelling Trinomials as Binomial Products	157
3.5 Polynomials of the Form $x^2 + bx + c$	159
3.6 Polynomials of the Form $ax^2 + bx + c$	168
Checkpoint 2	179
3.7 Multiplying Polynomials	182
3.8 Factoring Special Polynomials	188
Study Guide and Review	196
Practice Test	201

4 Roots and Powers

4.1 Math Lab: Estimating Roots	204
4.2 Irrational Numbers	207
4.3 Mixed and Entire Radicals	213
Checkpoint 1	220
4.4 Fractional Exponents and Radicals	222
4.5 Negative Exponents and Reciprocals	229
Checkpoint 2	235
4.6 Applying the Exponent Laws	237
Study Guide and Review	244
Practice Test	249
Project: Algebra and Number <i>Human Calculators</i>	250
Cumulative Review: Chapters 1 – 4	252

5 Relations and Functions

5.1	Representing Relations	256
5.2	Properties of Functions	264
	Checkpoint 1	274
5.3	Interpreting and Sketching Graphs	276
5.4	Math Lab: Graphing Data	284
5.5	Graphs of Relations and Functions	287
	Checkpoint 2	298
5.6	Properties of Linear Relations	300
5.7	Interpreting Graphs of Linear Functions	311
	Study Guide and Review	324
	Practice Test	329

6 Linear Functions

6.1	Slope of a Line	332
6.2	Slopes of Parallel and Perpendicular Lines	344
	Checkpoint 1	352
6.3	Math Lab: Investigating Graphs of Linear Functions	354
6.4	Slope-Intercept Form of the Equation for a Linear Function	357
6.5	Slope-Point Form of the Equation for a Linear Function	365
	Checkpoint 2	375
6.6	General Form of the Equation for a Linear Relation	377
	Study Guide and Review	386
	Practice Test	391

7 Systems of Linear Equations

7.1	Developing Systems of Linear Equations	394
7.2	Solving a System of Linear Equations Graphically	403
7.3	Math Lab: Using Graphing Technology to Solve a System of Linear Equations	411
	Checkpoint 1	414
7.4	Using a Substitution Strategy to Solve a System of Linear Equations	416
7.5	Using an Elimination Strategy to Solve a System of Linear Equations	428
	Checkpoint 2	440
7.6	Properties of Systems of Linear Equations	442
	Study Guide and Review	450
	Practice Test	455

Project: Relations and Functions

<i>Exercise Mind and Body</i>	456
Cumulative Review: Chapters 1 – 7	458
Answers	462
Glossary	535
Index	543
Acknowledgments	547

Your Book at a Glance

This book organizes your grade 10 course into three major topics. By focusing on one topic at a time, you can:

- spend more time on new concepts
- develop deeper understanding
- improve your recall of math concepts and strategies
- make connections across topics

MEASUREMENT

1. Measurement

2. Trigonometry

Project: Ramp It Up!

Cumulative Review: Chapters 1 and 2

Apply and extend what you know from previous grades as you investigate and solve real-world problems involving measurement.

ALGEBRA AND NUMBERS

3. Factors and Products

4. Roots and Powers

Project: Human Calculators

Cumulative Review: Chapters 1 – 4

Extend your work with patterning, algebra, and number concepts as you develop tools for solving new types of problems.

RELATIONS AND FUNCTIONS

5. Relations and Functions

6. Linear Functions

7. Systems of Linear Equations

Project: Exercise Mind and Body

Cumulative Review: Chapters 1 – 7

Build on what you have learned about algebra to study graphs and explore patterns.

Projects after Chapters 2, 4, and 7 have you solve applied problems while you reinforce your learning.

Cumulative Reviews cover all the content up to that point in the book.

Chapter Opener

Each chapter is organized around a few key **Big Ideas** of mathematics.

Learning through the **Big Ideas**:

- lets you make sense of math topics
- helps you understand how the topics are related
- lets you learn more efficiently

Look for an illustration where the math of the chapter is applied. The caption describes the application.

3 Factors and Products

AERIAL PHOTO OF MANITOBA
The Dominion Land Survey divides much of western Canada into 1-mile square sections. This photo shows canola fields around Shoal Lake, located in western Manitoba.

BUILDING ON

- determining factors and multiples of whole numbers to 100
- identifying prime and composite numbers
- determining square roots of rational numbers
- adding and subtracting polynomials
- multiplying and dividing polynomials by monomials

BIG IDEAS

- Arithmetic operations on polynomials are based on the arithmetic operations on integers, and have similar properties.
- Multiplying and factoring are inverse processes, and a rectangle diagram can be used to represent them.

NEW VOCABULARY

prime factorization
greatest common factor
least common multiple
perfect cube, cube root
factoring by decomposition
perfect square trinomial
difference of squares
radicand, radical, index



Building on...

tells you what you need to know before learning new concepts.

Big Ideas...

tell you the learning goals for the chapter.

New Vocabulary...

identifies the new terms you will use as you work through the chapter.

Numbered Lessons

Each lesson links to the **Big Ideas** stated at the start of the chapter.

Lesson Focus... states the learning goal for the lesson.

Make Connections... presents previous content, or an application, so you can make connections between what you already know, and what you are about to learn.

2.1 The Tangent Ratio

LESSON FOCUS
Deriving the tangent ratio and relating it to the angle of inclination of a line segment.

This ranger's cabin on Hinchinbrook Island, Tullaroona, has solar panels on its roof.



Make Connections

South-facing solar panels on a roof work best when the angle of inclination of the roof, that is, the angle between the roof and the horizontal, is approximately equal to the latitude of the house.

When an architect designs a house that will have solar panels on its roof, she has to determine the width and height of the roof so that the panels work efficiently.

What happens to the angle of inclination if the diagram of the house is drawn using a different scale?



You will investigate the relationship between one acute angle in a right triangle and two sides of that triangle.

70 Chapter 2: Trigonometry

In each lesson you **Construct Understanding**, then apply what you have learned.

Try This or Think About It... presents an activity or a problem that uses ideas from **Make Connections**. The activity or problem leads you to new concepts.

Construct Understanding

Recall that two triangles are similar if one triangle is an enlargement or a reduction of the other.


TRY THIS

Work with a partner.

You will need grid paper, a ruler, and a protractor.

- On grid paper, draw a right $\triangle ABC$ with $\angle B = 90^\circ$.
- Each of you draws a different right triangle that is similar to $\triangle ABC$.
- Measure the sides and angles of each triangle. Label your diagrams with the measures.
- The two shorter sides of a right triangle are its legs. Calculate the ratios of the legs $\frac{CB}{CA}$ as a decimal, then the corresponding ratios for each of the similar triangles.
- How do the ratios compare?
- What do you think the value of each ratio depends on?

We name the sides of a right triangle in relation to one of its acute angles.



The ratio $\frac{\text{Length of side opposite } \angle A}{\text{Length of side adjacent to } \angle A}$ depends only on the measure of the angle, not on how large or small the triangle is.

This ratio is called the **tangent ratio** of $\angle A$. The tangent ratio for $\angle A$ is written as $\tan A$. We usually write the tangent ratio as a fraction.

The Tangent Ratio


If $\angle A$ is an acute angle in a right triangle, then
$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

2.1 The Tangent Ratio 71

Colour boxes highlight important rules, formulas, or definitions.

2.2 Using the Tangent Ratio to Calculate Lengths

LESSON FOCUS
Using the tangent ratio to calculate lengths.



Make Connections

In Lesson 2.1, you used the measures of two legs of a right triangle to calculate the measures of the acute angles of the triangle. When you know the length of one leg of a right triangle and the measure of one acute angle, you can draw the triangle.

What other measures in the triangle can you calculate?

Construct Understanding

THINK ABOUT IT

Work with a partner.

In right $\triangle PQR$, $\angle Q = 90^\circ$, $\angle P = 34.3^\circ$, and $PQ = 46.1$ cm. Determine the length of RQ to the nearest tenth of a centimetre.

We use **direct measurement** when we use a measuring instrument to determine a length or an angle of a polygon. We use **indirect measurement** when we use mathematical reasoning to calculate a length or an angle.

The tangent ratio is a powerful tool we can use to calculate the length of a leg of a right triangle. We are then measuring the length of a side of a triangle **indirectly**. In a right triangle, we can use the tangent ratio, **opposite**, to write an equation. When we know the measure of an acute angle and the length of a leg, we solve the equation to determine the length of the other leg.

78 Chapter 2: Trigonometry

Look for margin notes that define or explain a key term.

Examples...
model strategies for solving problems.

Check Your Understanding
gives you an opportunity for immediate reinforcement after each **Example**.

Example 1 Determining the Length of a Side Opposite to a Given Angle

Determine the length of AB to the nearest tenth of a centimetre.

SOLUTION

In right $\triangle ABC$, AB is the side opposite $\angle C$ and BC is the side adjacent to $\angle C$.

Use the tangent ratio to write an equation.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

$$10 \tan 20^\circ = \frac{AB}{10}$$

Solve this equation for AB.

$$10 \tan 20^\circ = \frac{AB}{10}$$

$$10 \tan 20^\circ = \frac{AB}{10}$$

$$AB = 3.7725\dots$$

AB is approximately 3.8 cm long.

CHECK YOUR UNDERSTANDING

- Determine the length of XY to the nearest tenth of a centimetre.

How can you determine the length of the hypotenuse in $\triangle ABC$?

Example 2 Determining the Length of a Side Adjacent to a Given Angle

Determine the length of EF to the nearest tenth of a centimetre.

SOLUTIONS

Method 1

In right $\triangle DEF$, DE is opposite $\angle F$ and EF is adjacent to $\angle F$.

$$\tan F = \frac{\text{opposite}}{\text{adjacent}}$$

CHECK YOUR UNDERSTANDING

- Determine the length of YX to the nearest tenth of a centimetre.

How can you determine the length of YX?

2.2 Using the Tangent Ratio to Calculate Lengths 79

Example 3 Using Tangent to Solve an Indirect Measurement Problem

A searchlight beam shines vertically on a cloud. At a horizontal distance of 200 m from the searchlight, the angle between the ground and the line of sight to the cloud is 75° . Determine the height of the cloud to the nearest metre.

SOLUTION

Sketch and label a diagram to represent the information in the problem.

Assume the ground is horizontal.

In right $\triangle SCP$, side CS is opposite $\angle P$ and SP is adjacent to $\angle P$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 75^\circ = \frac{p}{200}$$

Solve the equation for p.

$$200 \tan 75^\circ = \frac{p}{200}$$

$$200 \tan 75^\circ = p$$

$$p = 933.0127\dots$$

The cloud is approximately 933 m high.

CHECK YOUR UNDERSTANDING

- At a horizontal distance of 200 m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 6° . How high is the tower to the nearest metre? The diagram is not drawn to scale.

What is the advantage of solving the equation for EF before calculating $\tan 20^\circ$?

Which method to determine EF do you think is easier? Why?

How could you determine the length of DF?

Discuss the Ideas

- How can you use the tangent ratio to determine the length of a leg in a right triangle?
- Suppose you know or can calculate the lengths of the legs in a right triangle. Why can you always calculate its hypotenuse?

2.2 Using the Tangent Ratio to Calculate Lengths 81

Margin questions extend your thinking, or ask you to think about key points.

Discuss the Ideas...
after working through the Examples, and before starting the Exercises.

Each lesson provides practice for the concepts you've been working with.

Exercises...
are organized by A/B/C levels of difficulty and allow you to check your skills and understanding.

Discuss the Ideas

- How do you determine the surface area of a right pyramid?
- When you use a picture of a right pyramid with a regular polygon base, how do you identify its height and its slant height?
- How is calculating the surface area of a right pyramid like calculating the surface area of a right cone? How is it different?

Exercises

A

- Determine the lateral area of each right pyramid to the nearest square unit.
 - square pyramid
 - regular tetrahedron
- Determine the surface area of each right pyramid in question 4, to the nearest square unit.
- Determine the lateral area of each right cone to the nearest square unit.
 - right cone
 - right cone
- Determine the surface area of each object to the nearest square unit.
 - right square pyramid
 - right cone

B

- The slant height of a right square pyramid is 7.0 ft, and the side length of the base is 4.0 ft.
 - Sketch the pyramid.
 - Determine its lateral area to the nearest square foot.
- The Great Pyramid at Giza has a square base with side length 750 ft, and an original height of 481 ft. Determine its original surface area to the nearest square foot.
- Aiden built a cone-shaped volcano for a school science project. The volcano has a base diameter of 32 cm and a slant height of 45 cm.
 - What is the lateral area of the volcano to the nearest tenth of a square centimetre?
 - The paint for the volcano's surface costs \$1.09/litre, and one jar of paint covers 400 cm². How much will the paint cost?
- A road pylon approximates a right cone with perpendicular height 53 cm and base diameter 18 cm. The lateral surface of the pylon is to be painted with reflective paint. What is the area that will be painted? Answer to the nearest square centimetre.
- Determine the surface area of each right pyramid to the nearest square unit.
 - right pyramid
 - right pyramid

C

- The Royal Saskatchewan Museum in Regina has a tipi in its First Nations Gallery. The tipi approximates a cone with a base diameter of 3.0 m and a height of 4.0 m. A Cow woman from Chick Lake tanned, prepared, and sewed 12 brown hides to make the cones. To the nearest tenth of a square metre, what area did each brown hide cover? What assumptions did you make?
- A farmer unloaded grain onto a tarp on the ground. The grain formed a cone-shaped pile that had a diameter of 12 ft and a height of 8 ft. Determine the surface area of the exposed grain to the nearest square foot.
- For each object, its surface area, SA, and some dimensions are given. Calculate the dimension indicated by the variable to the nearest tenth of a unit.
 - right cone
 - right square pyramid
- A my block must wooden block. One block is a right length of 2 in. and a second block is a right height of $\frac{3}{4}$ in. A third block is with base dimension of 3 in.
 - When the block is tall?
 - Which block?
- An igloo approximates a hemisphere, with an entrance tunnel that approximates half a right cylinder.
 - One model forms a sculpture that is a composite object comprising a right cylinder with base diameter 15 in. and height 3 in., and a right cone with the same base diameter as the base of the cylinder and a height of 6 in. Determine the volume of the sculpture to the nearest cubic inch.
 - The sculpture is part a carved out of a block of ice with the shape of a right square prism. What are the least possible dimensions for the prism to the nearest inch?
 - The sculpture is part a carved from a block of ice with the shape of a right rectangular prism with dimensions 16 in. by 15 in. by 12 in. What volume of ice, in cubic inches, remains?

Reflect

Which do you find easier to calculate the surface area of a composite object or its volume? Explain your choice.

THE WORLD OF MATH

Profile: Festival du Voyageur

The Festival du Voyageur is an annual event that takes place in Winnipeg every February to celebrate the city's Francophone and Métis cultural heritage. Major attractions at the festival are the snow sculptures that are displayed at Voyageur Park and in neighbourhoods around the city. The festival also includes an International Snow Sculpting Symposium, where teams of sculptors create unique artwork from blocks of snow measuring 3.0 m by 3.7 m by 3.7 m. Each year, sculptors transform 450 000 cubic feet of snow into a winter wonderland.

What is the volume of snow in a sculpture that measures 50 ft by 18 ft by 6 ft?

1.7 Solving Problems Involving Objects 61

Reflect...
prompts you to think about, and record, what you learned.

The World of Math...
highlights interesting math facts from the world around you, from history, or from the world of careers.

Math Lab Lessons


Math Lab lessons provide more time to explore the math using materials or technology.

As in other lessons, you'll find the **Lesson Focus** and **Make Connections**.

Try This... presents an extended activity.

2.3 MATH LAB
Measuring an Inaccessible Height

LESSON FOCUS
Determine a height that cannot be measured directly.



Make Connections
The farmers use a clinometer to measure the angle between a horizontal line and the line of sight to the top of a tree. They measure the distance to the base of the tree. How can they use the tangent ratio to calculate the height of the tree?


Construct Understanding

TRY THIS
Work with a partner.
You will need:

- an enlarged copy of a 180° protractor
- scissors
- a measuring tape or 2 metre sticks
- a piece of heavy cardboard big enough for you to attach the paper protractor
- a drinking straw
- glue
- adhesive tape
- a needle and thread
- a small metal washer or weight
- grid paper


A. Make a drinking straw clinometer:

- Glue or tape the paper protractor to the cardboard. Carefully cut it out.
- Use the needle to pull the thread through the cardboard at the centre of the baseline of the protractor. Secure the thread to the back of the cardboard with tape. Attach the weight to the other end of the thread.
- Tape the drinking straw along the baseline of the protractor for use as a sighting tube.



B. With your partner, choose a tall object whose height you cannot measure directly (for example, a flagpole, a tennis pole, a tree, or a building).
One of you stands near the object on level ground.
Your partner measures and records your distance from the object.

D. Hold the clinometer as shown, with the weight hanging down.



After completing **Try This**, check if you're on the right track.

Assess Your Understanding... provides a few key questions so you can apply what you learned in **Try This**.

How does the acute angle between the thread and the straw relate to the angle of inclination of the straw?

What other strategy could you use to determine the height of the object?

Keep your clinometer for use in the Review.

E. Look at the top of the object through the straw. Your partner records the acute angle indicated by the thread on the protractor.

F. Your partner measures and records how far your eye is above the ground.

G. Sketch a diagram with a vertical line segment representing the object you want to measure. Label:

- your distance from the object
- the vertical distance from the ground to your eyes
- the angle of inclination of the straw

H. Change places with your partner. Repeat Steps B to G.

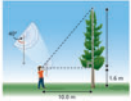
I. Use your measurements and the tangent ratio to calculate the height of the object.

J. Compare your results with those of your partner. Does the height of your eye affect the measurements? The final result? Explain.

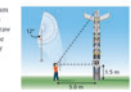
Assess Your Understanding

1. Explain how the angle shown on the protractor of your clinometer is related to the angle of inclination that the clinometer measures.

2. A tree farmer stood 10.0 m from the base of a tree. She used a clinometer to sight the top of the tree. The angle shown on the protractor scale was 49°. The tree farmer held the clinometer 1.0 m above the ground. Determine the height of the tree to the nearest tenth of a metre. The diagram is not drawn to scale.



3. Use the information in the diagram to calculate the height of a tennis pole observed with a drinking-straw clinometer. Give the answer to the nearest metre. The diagram is not drawn to scale.



Review and Study Features

Checkpoints occur at key intervals in the chapter, so you can reflect on Big Ideas as they've been developed. Checkpoints let you check your understanding so far.

CHECKPOINT 1

Connections	Concept Development	Assess Your Understanding																																										
<p>Connections... gives a picture of what you've been learning.</p>	<p>Concept Development... summarizes important content.</p> <p>In Lesson 3.1</p> <ul style="list-style-type: none"> You use whole number properties and operations to determine prime factors. You use prime factors to determine greatest common factor (GCF) and least common multiple (LCM). <p>In Lesson 3.2</p> <ul style="list-style-type: none"> You use factors and multiples to determine perfect square whole numbers and their square roots. You use factors and multiples to determine perfect cube whole numbers and their cube roots. 	<p>Assess Your Understanding</p> <p>3.1</p> <ol style="list-style-type: none"> Use properties of prime factors for each number as a product of its prime factors. <table border="0" style="width: 100%;"> <tr> <td>a) 1200</td> <td>b) 4224</td> <td>c) 4120</td> </tr> <tr> <td>d) 1043</td> <td>e) 3024</td> <td>f) 3075</td> </tr> </table> Determine the greatest common factor of each set of numbers. <table border="0" style="width: 100%;"> <tr> <td>a) 45, 48, 54</td> <td>b) 84, 120, 144</td> </tr> <tr> <td>c) 145, 205, 290</td> <td>d) 200, 300, 520</td> </tr> <tr> <td>e) 450, 1200, 360</td> <td>f) 950, 1225, 1350</td> </tr> </table> Determine the least common multiple of each set of numbers. <table border="0" style="width: 100%;"> <tr> <td>a) 12, 15, 20</td> <td>b) 12, 20, 32</td> <td>c) 18, 24, 30</td> </tr> <tr> <td>d) 30, 32, 40</td> <td>e) 48, 56, 64</td> <td>f) 50, 55, 60</td> </tr> </table> Use the least common multiple to help determine each answer. <table border="0" style="width: 100%;"> <tr> <td>a) $\frac{4}{5} + \frac{3}{11}$</td> <td>b) $\frac{12}{5} - \frac{4}{7}$</td> <td>c) $\frac{12}{10} \cdot \frac{7}{3}$</td> </tr> </table> <p>3.2</p> <ol style="list-style-type: none"> Determine the square root of each number. Which different strategies could you use? <table border="0" style="width: 100%;"> <tr> <td>a) 400</td> <td>b) 784</td> <td>c) 576</td> </tr> <tr> <td>d) 1089</td> <td>e) 1521</td> <td>f) 3025</td> </tr> </table> Determine the cube root of each number. Which different strategies could you use? <table border="0" style="width: 100%;"> <tr> <td>a) 1728</td> <td>b) 3375</td> <td>c) 4000</td> </tr> <tr> <td>d) 3002</td> <td>e) 10488</td> <td>f) 2164</td> </tr> </table> Determine whether each number is a perfect square, a perfect cube, or neither. <table border="0" style="width: 100%;"> <tr> <td>a) 2808</td> <td>b) 3136</td> <td>c) 4096</td> </tr> <tr> <td>d) 4024</td> <td>e) 5832</td> <td>f) 9270</td> </tr> </table> Between each pair of numbers, identify all the perfect squares and perfect cubes that are whole numbers. <table border="0" style="width: 100%;"> <tr> <td>a) 400 - 500</td> <td>b) 900 - 1000</td> <td>c) 1100 - 1175</td> </tr> </table> <p>10. A cube has a volume of 2197 m^3. Its surface is to be painted. Each can of paint covers about 40 m^2. How many cans of paint are needed? Justify your answer.</p>	a) 1200	b) 4224	c) 4120	d) 1043	e) 3024	f) 3075	a) 45, 48, 54	b) 84, 120, 144	c) 145, 205, 290	d) 200, 300, 520	e) 450, 1200, 360	f) 950, 1225, 1350	a) 12, 15, 20	b) 12, 20, 32	c) 18, 24, 30	d) 30, 32, 40	e) 48, 56, 64	f) 50, 55, 60	a) $\frac{4}{5} + \frac{3}{11}$	b) $\frac{12}{5} - \frac{4}{7}$	c) $\frac{12}{10} \cdot \frac{7}{3}$	a) 400	b) 784	c) 576	d) 1089	e) 1521	f) 3025	a) 1728	b) 3375	c) 4000	d) 3002	e) 10488	f) 2164	a) 2808	b) 3136	c) 4096	d) 4024	e) 5832	f) 9270	a) 400 - 500	b) 900 - 1000	c) 1100 - 1175
a) 1200	b) 4224	c) 4120																																										
d) 1043	e) 3024	f) 3075																																										
a) 45, 48, 54	b) 84, 120, 144																																											
c) 145, 205, 290	d) 200, 300, 520																																											
e) 450, 1200, 360	f) 950, 1225, 1350																																											
a) 12, 15, 20	b) 12, 20, 32	c) 18, 24, 30																																										
d) 30, 32, 40	e) 48, 56, 64	f) 50, 55, 60																																										
a) $\frac{4}{5} + \frac{3}{11}$	b) $\frac{12}{5} - \frac{4}{7}$	c) $\frac{12}{10} \cdot \frac{7}{3}$																																										
a) 400	b) 784	c) 576																																										
d) 1089	e) 1521	f) 3025																																										
a) 1728	b) 3375	c) 4000																																										
d) 3002	e) 10488	f) 2164																																										
a) 2808	b) 3136	c) 4096																																										
d) 4024	e) 5832	f) 9270																																										
a) 400 - 500	b) 900 - 1000	c) 1100 - 1175																																										

148 Chapter 3: Factors and Products Checkpoint 1 149

Connections...
gives a picture of what you've been learning.

Concept Development...
summarizes important content.

Assess Your Understanding...
provides questions related to each lesson.

At the end of each chapter:

Study Guide...
summarizes the important concepts and skills from the chapter.

STUDY GUIDE

CONCEPT SUMMARY	SKILLS SUMMARY
<p>Big Ideas</p> <ul style="list-style-type: none"> Any number that can be written as the fraction $\frac{a}{b}$, $a \neq 0$, where a and b are integers, is rational. Exponents can be used to represent roots and reciprocals of rational numbers. The exponent laws can be extended to include powers with rational and variable bases, and rational exponents. <p>Applying the Big Ideas</p> <ul style="list-style-type: none"> This means that: <ul style="list-style-type: none"> If a real number can be expressed as a terminating or repeating decimal, it is rational; otherwise, it is irrational. The numerator of a rational exponent indicates a power, while the denominator indicates a root. A negative exponent indicates a reciprocal. We can use the exponent laws to simplify expressions that involve rational exponents. <p>Reflect on the Chapter</p> <ul style="list-style-type: none"> How can you predict whether the value of a radical will be a rational number or an irrational number? How were the exponent laws used to create definitions for negative exponents and rational exponents? What does it mean to simplify an expression involving radicals or exponents? 	<p>Classify numbers. [4.1, 4.2]</p> <ul style="list-style-type: none"> To determine whether a number is rational or irrational, write the number in decimal form. <ul style="list-style-type: none"> Repeating and terminating decimals are rational. Non-repeating, non-terminating decimals are irrational. <p>Simplify radicals. [4.3]</p> <ul style="list-style-type: none"> To simplify a square root: <ol style="list-style-type: none"> Write the radicand as a product of its greatest perfect square factor and another number. Take the square root of the perfect square factor. A similar procedure applies for cube roots and higher roots. <p>Evaluate powers. [4.4, 4.5]</p> <ul style="list-style-type: none"> To evaluate powers without using a calculator: <ol style="list-style-type: none"> Rewrite a power with a negative exponent as a power with a positive exponent. Represent powers with fractional exponents as radicals.

Review...
pages provide additional practice.

REVIEW

4.1

- Evaluate each radical. Why do you not need a calculator?

a) $\sqrt{1000}$	b) $\sqrt{1001}$
c) $\sqrt{54}$	d) $\sqrt{423}$
- Explain, using examples, the meaning of the index of a radical.
- Estimate the value of each radical to 1 decimal place. What strategies can you use?

a) $\sqrt{11}$	b) $\sqrt{-12}$	c) $\sqrt{15}$
----------------	-----------------	----------------
- Identify the number in each case:
 - 3 is a square root of the number.
 - 16 is the cube root of the number.
 - $\sqrt[4]{7}$ is a fourth root of the number.
- For $\sqrt{35}$, does its decimal form terminate, repeat, or neither? Support your answer with an explanation.

4.2

- For **each** number, is it rational or irrational. Justify your answer.

a) -2	b) 17	c) $\sqrt{16}$
d) $\sqrt{32}$	e) 0.256	f) 12.3
g) π	h) $\sqrt{21}$	i) π
- Determine the approximate side length of a square with area 23 cm^2 . How could you check your answer?

Look at this calculator screen.

a) Is the number 3.311 392 454 rational or irrational? Explain.
b) Is the number π rational or irrational? Explain your answer.
- Place each number on a number line, then order the numbers from least to greatest.

$\sqrt{36}$, 25 , $\sqrt{16}$, $\sqrt{-36}$, 36 , $\sqrt{16}$
--

4.3

- The formula $V = 2s^2 \sqrt{\frac{3}{2}}$ gives the volume, V , in cubic feet, for one complete revolution of a pendulum with length s metres. If the pendulum is 0.25 m long. What is the pendulum's rate to complete one swing? Give the answer to the nearest second.

4.4

- Write each radical in simplest form.

a) $\sqrt{72}$	b) $\sqrt{180}$
c) $\sqrt{132}$	d) $\sqrt{150}$
- Write each mixed radical as an entire radical.

a) $6\sqrt{3}$	b) $3\sqrt{14}$
c) $4\sqrt{3}$	d) $2\sqrt{2}$
- Alfalfa cubes are fed to horses to provide protein, minerals, and vitamins.

Two sizes of cubes have volumes 32 cm^3 and 11 cm^3 . What is the difference in the edge lengths of the cubes? How can you use radicals to find out?

PRACTICE TEST

For questions 1 and 2, choose the correct answer: A, B, C, or D.

- The volume V cubic inches of each cube is given. For which cube is the edge length an irrational number?

<p>A. </p> <p>C. </p>	<p>B. </p> <p>D. </p>
-----------------------	-----------------------
- Which number is rational?

A. $\sqrt{100}$	B. $\sqrt{30}$	C. $\sqrt{\frac{22}{11}}$	D. π
-----------------	----------------	---------------------------	----------
- a) Which is greater, $\sqrt{70}$ or $\sqrt{37}$? Justify your answer.
b) Sketch a number line to illustrate the numbers in part a.
- Evaluate without using a calculator:

a) $\sqrt{720}$	b) $(-4)^{-2}$	c) $64^{\frac{1}{3}}$	d) $16^{-\frac{1}{2}}$
-----------------	----------------	-----------------------	------------------------
- Write $4^{\frac{1}{2}}$ as a radical in simplest form.
A student simplified $\frac{2\sqrt{3}}{\sqrt{2}}$ as follows:
 $\frac{2\sqrt{3}}{\sqrt{2}} = \frac{2\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{6}}{2} = \sqrt{6}$
Is the student correct? If not, describe any errors and write a correct solution.
- Simplify each expression. Write your answers using positive exponents.

a) $(2^{\frac{1}{2}})^3 (2^{\frac{1}{2}})^2$	b) $(\frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}})^2$
--	--
- Scientists use the formula $d = 0.099m^3$ to calculate the volume of water, d , in litres, that a mammal with mass m kilograms should drink in 1 day. Calculate how much water a 550-kg moose should drink in one day.

Practice Test 249

Practice Test...
lets you try a sample test before you take a class test.

Cumulative Reviews and Projects

After 2 or 3 chapters, these book features support your learning.

Cumulative Reviews...
help you recall content from earlier in the course.

CUMULATIVE REVIEW Chapters 1 and 2

- Andrea is constructing a pen for her dog. The perimeter of the pen is 70 ft.
 - What is the perimeter of the pen in yards and feet?
 - The fencing material is sold by the yard. It costs \$2.09/yard. What will be the cost of this material before tax?
- A map of Alberta has a scale of 1:4 250 000. The map distance between Edmonton and Calgary is 6.5 cm. What is the distance between the two cities in the nearest kilometre?
- Describe how you would determine the radius of a cylindrical pipe in both imperial units and SI units.
- Convert each measurement.
 - 7 yd to the nearest centimetre
 - 11 000 000 in. to the nearest metre
 - 3 km to the nearest mile
 - 100 cm to feet and the nearest inch
- On the Alex Fraser Bridge in Delta, B.C., the maximum height of the road above the Fraser River is 134 m. On the Tacoma Narrows Bridge in Tacoma, Washington, the maximum height of the road above The Narrows is 109 ft. Which road is higher above the water? How much higher is it?
- Determine the surface area of each object to the nearest square unit.
 - regular tetrahedron
 - right cone
- The height of a right square pyramid is 40 in., and the side length of the base is 48 in. Determine the lateral area of the pyramid to the nearest square inch.
- The base of a hemisphere has a circumference of 36.3 mm. Determine the surface area and volume of the hemisphere to the nearest tenth of a millimetre.
- Determine the volume of the cone in question 6, to the nearest cubic unit.
- The diameter of the base of a right cone is 12 yd, and the volume of the cone is 216 cubic yards. Describe how to calculate the height of the cone to the nearest yard.
- One right square pyramid has base side length 10 cm and height 8 cm. Another right square pyramid has base side length 8 cm and height 13 cm. Does the pyramid with the greater volume also have the greater surface area? Justify your answer.
- A hemisphere has radius 20 in. A sphere has radius 17 in.
 - Which object has the greater surface area? How much greater is it to the nearest square inch?
 - Which object has the greater volume? How much greater is it to the nearest cubic inch?
- This composite object is a rectangular pyramid on top of a rectangular prism. Determine the surface area and volume of the composite object to the nearest unit.
- The height of a right square pyramid is 40 in., and the side length of the base is 48 in. Determine the lateral area of the pyramid to the nearest square inch.
- The base of a hemisphere has a circumference of 36.3 mm. Determine the surface area and volume of the hemisphere to the nearest tenth of a millimetre.
- Determine the angle of inclination of each line AB to the nearest tenth of a degree.
 -
 -
- Barry is collecting data on the heights of trees. He measures a horizontal distance of 20 yd from the base of a tree. Barry lies on the ground at this point and uses a clinometer to measure the angle of elevation of the top of the tree as 32°. Determine the height of the tree to the nearest yard.
- Ivy Cochran walked a right-angled path between the Niagara Fallsview Hotel and the Skyline Tower in Niagara Falls. He began at the hotel. The rope dipped upward and the average angle between the rope and the horizontal was 4°. Ivy walked a horizontal distance of 1700 ft. To the nearest foot, determine the vertical distance he travelled.
- Determine the measure of each indicated angle to the nearest tenth of a degree.
 -
 -
- A 12-ft ladder leans against a wall. The base of the ladder is 4 ft from the wall. To the nearest degree, what is the measure of the angle between the ladder and the wall?
- A basketball court is rectangular with diagonal length approximately 100 ft. The angle between a diagonal and a longer side is 30°. Determine the dimensions of the basketball court to the nearest foot.
- Solve each right triangle. Write the measures to the nearest tenth.
 -
 -
- A helicopter is 15 km due east of its base when it receives a call to pick up a stranded snowboarder. The person is on a mountain 9 km due south of the helicopter's present location. When the helicopter picks up the snowboarder, what is the measure of the angle between the path the helicopter took flying south and the path it will take to fly directly to its base? Write the angle to the nearest degree.
- Determine the length of each indicated side and the measures of all the angles in this diagram. Write the measures to the nearest tenth.

130 Chapters 1 and 2 Cumulative Review 131

Projects...
present applications where you can use the math you have learned to solve problems.

PROJECT MEASUREMENT

Ramp It Up!

New public buildings should be accessible by all, and if entry involves steps, an access ramp must be provided. Older buildings are often retro-fitted with ramps to provide easy access.

PART B: COST ESTIMATES

A ramp may be constructed from wood or concrete.

- Research, then estimate, the cost of constructing your ramp using lumber from your local supplier. Assume that the ramp surface will be asphaltboard, or particleboard, or plywood. Give details of all the connections between systems of measures.
- Research, then estimate, the cost of constructing your ramp using concrete. Outdoor ramps can become slippery in wet or cold weather, so your ramp surface should have a non-slip coating.
- Research, then estimate the cost of covering the ramp surface with non-slip paint or other material.

PROJECT PRESENTATION

Your completed project should include:

- Your diagrams, with calculations and explanations to support your design.
- Cost estimates for the construction in both wood and concrete, with all supporting calculations.

PROJECT ALGEBRA AND NUMBER

Human Calculators

Throughout history, there have been men and women so proficient in calculating mentally that they have been called "human calculators."

In 1980, Shakuntala Devi mentally multiplied the numbers 9 689 304 870 870 and 2 465 099 145 739 and gave the correct answer 18 942 668 177 995 426 462 773 730 in 28 s.

In 2004, Alexis Lemaire found the 13th root of a 100-digit number in less than 4 s. In 2007, he was able to find the 13th root of a 200-digit number in a little over 1 min.

PART B: INVESTIGATING OTHER LINEAR RELATIONS

- Use some research about other physical activities that are not listed in the table in Part A. Determine the rate at which energy is used in Calories per unit time.
- Conduct investigations, create situations, then pose problems that involve linear relations or functions. Linear systems: graphing points of intersection; and so on.

PROJECT PRESENTATION

Your completed project can be presented in a written or oral format but should include:

- A list of investigations you conducted, the situations you created, and the problems you posed, along with explanations of your strategies for solving the problems.
- A display of the graphs that you made, including an explanation of how and why you created them and how you interpreted them.

EXTENSION

Because exercise and nutrition play an important role in the health of people, much research has been conducted in these areas. Many linear relations have been discovered during these studies.

- Investigate other linear relations related to exercise, nutrition, and physical activities.
- Identify different types of energy and how they are measured. You might use an internet search using key words such as joules, kilojoules, calories, and kilocalories.
- Write a brief report on the linear relations you discovered and how you know they are linear.

PART B: INVESTIGATING MENTAL CALCULATION METHODS

Investigate your own methods or research to find mental math methods that can be used to:

- Square different types of 2-digit numbers.
- Calculate the square root of 8-digit square numbers such as 2011.

PROJECT PRESENTATION

Your completed project can be presented in a written or oral format but should include:

- An explanation of your methods with examples.
- An explanation of why the method works: use algebra, number patterns, diagrams, or models such as algebra tiles to support your explanation. You may need to research to find the explanation.

EXTENSION

Because most people do not have extraordinary mental calculation abilities, relatively complex written methods were invented to calculate or estimate roots.

- Use an internet search or examine older math textbooks to identify some methods or formulas that have been used to calculate or estimate roots, particularly square roots and cube roots. These might include formulas developed by Newton, Heron, and Bhaskara.
- Provide a brief written report with an example of how to use one of these methods. Try to explain why the method works.

The Bakhshali manuscript was found in Pakistan in 1881. It is believed to be a 7th century copy of a manuscript written in the 3rd century or earlier.

456 Relations and Functions Project 457

PROJECT RELATIONS AND FUNCTIONS

Exercise Mind and Body

Cross training involves varying the types of exercises you do in each workout, to use different muscles and different amounts of energy. For example, you might run and lift weights in one workout, then swim in the next workout.

Stationary bike energy used = 420 Calories per hour
Swimming energy used = 480 Calories per hour

PART A: INVESTIGATING RELATIONS

This table shows the approximate rate at which energy is used, in Calories per hour, for three physical activities. The rate at which energy is used is related to the mass of the person doing the activity.

Activity	Mass (kg)	Rate of Energy Use (Cal/h)	Mass (kg)	Rate of Energy Use (Cal/h)	Mass (kg)	Rate of Energy Use (Cal/h)	Mass (kg)	Rate of Energy Use (Cal/h)
Stationary bike	50	350	60	420	70	490	80	560
Swimming	50	400	60	480	70	560	80	640
Walking uphill	50	300	60	360	70	420	80	480

Choose one physical activity from the table. Write five ordered pairs for this activity. Graph the ordered pairs. Use graphing technology if it is available. Is there a linear relationship between mass and the rate at which energy is used? How do you know?

456 Relations and Functions Project 457

